Department of Basic Science **Level: 1** **Examiner: Dr. Mohamed Eid** **Time allowed: 3 hours** **Answer all questions**	Pyramids higher Institute P.H.I. For Engineering And Technology معهد الأهرامات العالى للهندسة و التكنولوجيا	**Prep. Year: Final Exam** **Course: Mathematics 2** **Course Code: BAS 013 B** **Date: January 21, 2016**
The Exam consists of one page	**No. of questions: 5**	**Total Mark:** 70

Question 1

Find **y`** from the following: 18

(a) $y = 3^{2x} + \tanh 3x$ (b) $y = x^4 . \cosh x^2$

(c) $\mathrm{y} = 3 + \ln \mathrm{x} + \log \mathrm{x}$ (d) $y = \tan^{-1} x + \sin^{-1}\mathrm{x}$

(e) $\mathrm{y} = \mathrm{t} + \ln \mathrm{t}\ ,\ \mathrm{x} = \mathrm{t}.\mathrm{e}^{\mathrm{t}}$ (f) $\mathrm{y}^5 = \mathrm{x}^3 + \sin(\mathrm{xy})$

Question 2

Find the following integrals: 18

(i) $\int (x^4 + 2^{3x} + 3)dx$ (ii) $\int (\frac{x}{1+x^2} + \frac{2x}{\sqrt{3+x^2}})dx$

(iii) $\int (x + \frac{1}{x})^2\, dx$ (iv) $\int (\frac{1}{3} + \frac{2}{\mathrm{x}} + \frac{1}{\mathrm{x}+1})\mathrm{dx}$

(v) $\int 3x^2(3 + x^3)^8\, dx$ (vi) $\int \ln x\, dx$

(vii) $\int \sin 3\mathrm{x} . \cos \mathrm{x}\, \mathrm{dx}$ (viii) $\int \cos^3 \mathrm{x}\, \mathrm{dx}$ (ix) $\int \frac{2x-1}{x^2-4x+3} dx$

Question 3

(a)Find the area of the region between the curve $y = x^3 - x$, x-axis, x in [0, 2]. 4

(b)If the region between the curve $y = x + \frac{1}{x}$, x-axis, x in [1, 2] is rotated about : 8

(i) x-axis (ii)y-axis. Find the volume of the generated solids V_x , V_y .

Question 4

(a)State the definition of the plane. 2

(b)Find the angle between the planes: 2x – y + 2z + 7 = 0 , x + 2y + 2z = 0 4

(c)Write the equation of the plane that passes through the points: 6

(1, 1, 2), (–1, 1, 4), (3, 0, –1).

Question 5

(a)State the definition of the sphere. 2

(b)Write the equation of the plane that passes through the point (1, –1, 2) and its normal vector $\overline{\mathrm{N}} = 2\mathrm{i} - 3\mathrm{j} + \mathrm{k}$. 4

(c)Write the name of each surface:

(i) $x^2 + y^2 + z^2 - 3x + 2z = 0$ (iii) $2x^2 + z^2 = 3$	(ii) $y^2 - x^2 - 3z^2 = 0$ (iv) $y^2 + z^2 = 3x^2$	4

Good Luck *Dr. Mohamed Eid*

Answer

Answer of Question 1

(a) $y = 3^{2x} + \tanh 3x$ $\quad$ $y` = 9^x \ln 9 + 3\,\text{sech}^2\, 3x$

(b) $y = x^4 . \cosh x^2$ $\quad$ $y` = x^4 . \sinh x^2 \,.\, 2x + 4x^3 . \cosh x^2$

(c) $y = 3 + \ln x + \log x$ $\quad$ $y` = o + \frac{1}{x} + \frac{1}{\ln\, 10}\frac{1}{x}$

(d) $y = \tan^{-1} x + \sin^{-1} x$ $\quad$ $y` = \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}}$

(e) $y = t + \ln t\,,\;\; x = t.e^t$ $\quad$ $y` = \frac{\dot{y}}{\dot{x}} = \frac{1+\frac{1}{t}}{t.e^t + e^t}$

(f) $y^5 = x^3 + \sin(xy)$

Then $5y^5 . y` = 3x^2 + \cos(xy).(xy` + y)$

Then $y` = \frac{3x^2 + y\cos(xy)}{5y^5 - x\cos(xy)}$

--*18-Marks*

Answer of Question 2

(i) $\int (x^4 + 2^{3x} + 3)dx = \frac{x^5}{5} + \frac{8^x}{\ln 8} + 3x + c$

(ii) $\int (\frac{x}{1+x^2} + \frac{2x}{\sqrt{3+x^2}})dx = \int (\frac{1}{2}\frac{2x}{1+x^2} + 2x(3+x^2)^{-1/2})dx$

$$= \frac{1}{2}\ln(1+x^2) + 2(3+x^2)^{1/2} + c$$

(iii) $\int (x+\frac{1}{x})^2\,dx = \int (x^2+2+\frac{1}{x^2})\,dx = \frac{x^3}{3}+2x-\frac{1}{x}+c$

(iv) $\int (\frac{1}{3}+\frac{2}{x}+\frac{1}{x+1})dx = \frac{1}{3}x + 2\ln x + \ln(x+1) + c$

(v) $\int 3x^2(3+x^3)^8\,dx = \frac{1}{9}(3+x^3)^8 + c$

(vi) $\int \ln x\,dx$

By parts, $u = \ln x$, $dv = dx$, $u` = \frac{1}{x}$, $v = x$

Then $\int \ln x\,dx = uv - \int v\,du$

$$= x\ln x - \int 1\,dx = x\,ln\,x - x + c$$

(vii) $\int \sin 3x \,.\, \cos x\,dx = \frac{1}{2}\int (\sin 2x + \sin 4x)\,dx$

$$= -\frac{1}{4}\cos 2x - \frac{1}{8}\cos 4x + c$$

(viii) $\int \cos^3 x\,dx = \int \cos x \cos^2 x\,dx = \int \cos x\,(1-\sin^2 x)\,dx$

$$= \sin x - \frac{1}{3}\sin^2 x + c$$

(ix) $\int \frac{2x-1}{x^2-4x+3}dx$

By partial fractions, $\frac{2x-1}{x^2-4x+3} = \frac{A}{x-3} + \frac{B}{x-1} = \frac{A(x-1)+B(x-3)}{x^2-4x+3}$

Then $2x-1 = A(x-1)+B(x-3)$

Putting x = 3, then A = 5/2.

Putting x = 1, then B = – 1/2.

Then $\int \frac{2x-1}{x^2-4x+3}dx = \int (\frac{\frac{5}{2}}{x-3} - \frac{\frac{1}{2}}{x-1})\,dx = \frac{5}{2}\ln(x-3) - \frac{1}{2}\ln(x-1) + c$

--*18-Marks*

Answer of Question 3

(a)Solving the equation : $x^3 - x = 0$, we get x = 0, x = 1, x = –1.
We see that x = 1 lies inside the interval [0, 2].
Then, the area is :

$$A = \int_0^1 (x^3 - x)\, dx + \int_1^2 (x^3 - x)\, dx = \left[\frac{1}{4}x^4 - \frac{1}{2}x^2\right] + \left[\frac{1}{4}x^4 - \frac{1}{2}x^2\right]$$

$$A = \int_0^1 (x^3 - x)\, dx + \int_1^2 (x^3 - x)\, dx = \left[\frac{1}{4}x^4 - \frac{1}{2}x^2\right] + \left[\frac{1}{4}x^4 - \frac{1}{2}x^2\right]$$

$$= \left|-\frac{1}{4}\right| + \frac{9}{4} = \frac{10}{4} = \frac{5}{2}$$

---*4-Marks*

(b)(i)Rotation about x – axis:

$$V_x = \pi \int_1^2 f(x)^2\, dx = \pi \int_1^2 (x + \tfrac{1}{x})^2\, dx = \pi \int_1^2 (x^2 + 2 + \tfrac{1}{x^2})\, dx$$

$$= \pi \left[\frac{1}{3}x^3 + 2x - \frac{1}{x}\right] = \frac{29}{6}\pi$$

---*4-Marks*

(ii)Rotation about y – axis:

$$V_y = 2\pi \int_1^2 x\, f(x)\, dx = 2\pi \int_1^2 x(x + \tfrac{1}{x})\, dx = 2\pi \int_1^2 (x^2 + 1)\, dx$$

$$= 2\pi \left[\frac{1}{3}x^3 + x\right] = \frac{20}{3}\pi$$

---*4-Marks*

Answer of Question 4

(a)Definition of plane.

---*4-Marks*

(b)The normal vector of the first plane is : $\overline{N_1} = 2i - j + 2k$

The normal vector of the first plane is : $\overline{N_2} = i + 2j + 2k$

The angle between the planes is :

$$\cos\theta = \frac{\overline{N_1}.\overline{N_2}}{|\overline{N_1}|.|\overline{N_2}|} = \frac{2-2+4}{3x3} = \frac{4}{9}$$

--*4-Marks*

(c)The vector $\overline{P_1P_2} = -2i + 2k$ and the vector $\overline{P_1P_3} = 2i - j - 3k$.

The normal vector of the required plane is :

$$\overline{P_1P_2}x\overline{P_1P_3} = \begin{vmatrix} i & j & k \\ -2 & 0 & 2 \\ 2 & -1 & -3 \end{vmatrix} = 2i - 2j + 2k$$

Then the required plane is : 2(x – 1) – 2(y – 1) + 2(z – 2) = 0.

--*6-Marks*

<u>Answer of Question 5</u>

(a)Definition of sphere.

--*2-Marks*

(b)The required plane is : 2(x – 1) – 3(y + 1) + (z – 2) = 0.

--*4-Marks*

(c)(i) $x^2 + y^2 + z^2 - 3x + 2z = 0$ Sphere

(ii) $y^2 - x^2 - 3z^2 = 0$ Cone with y – axis

(iii) $2x^2 + z^2 = 3$ Cylinder with y – axis

(iv) $y^2 + z^2 = 3x^2$ Cone with x – axis

--*4-Marks*